

Deconfinement and universality in the 3D $U(1)$ lattice gauge theory at finite temperature: study in the dual formulation

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Abstract

We study analytically and numerically the three-dimensional $U(1)$ lattice gauge theory at finite temperature in the dual formulation. For an appropriate disorder operator, we obtain the renormalization group equations describing the critical behavior of the model in the vicinity of the deconfinement phase transition. These equations are used to check the validity of the Svetitsky-Yaffe conjecture regarding the critical behavior of the lattice $U(1)$ model. Furthermore, we perform numerical simulations of the model for $N_t = 1, 2, 4, 8$ and compute, by a cluster algorithm, the dual correlation functions and the corresponding second moment correlation length. In this way we locate the position of the critical point and calculate critical indices.

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1 Introduction

The Svetitsky-Yaffe conjecture [1] states that, if the correlation length of a D -dimensional finite-temperature gauge theory with a given symmetry group G diverges at the transition point, then the gauge theory belongs to the same universality class of the $(D - 1)$ -dimensional spin model possessing the center of the group G as global symmetry. This connection at criticality is relevant for a large class of gauge systems and turns out to be fundamental in understanding the deconfinement phase transition of heavy colored charges, *i.e.* pure gauge QCD.

The validity of the Svetitsky-Yaffe conjecture in presence of an infinite-order phase transition is yet to be verified. This is the case, for instance, of the deconfinement phase transition in the finite-temperature $3D$ $U(1)$ lattice gauge theory (LGT). If the Svetitsky-Yaffe conjecture holds, this theory should belong to the universality class of the $2D$ XY model, which is known to undergo a Berezinskii-Kosterlitz-Thouless (BKT) [2, 3] phase transition. This means that the two models should exhibit the same critical indices, thus implying a common dynamical behavior at criticality. In general, critical indices are extracted from the study of the dependence of the order parameter on the couplings of the theory near the transition point. In finite-temperature $3D$ $U(1)$ LGT, as well as in the $2D$ XY spin model, due to the Mermin-Wagner theorem [4], no spontaneous symmetry breaking can occur and, consequently, there exist no order parameter. A critical index can nevertheless be defined in $3D$ $U(1)$ through the correlation function of two Polyakov loops, which is the counterpart of the spin-spin correlator in the $2D$ XY model. Indeed this correlation function decreases with a power law for $\beta \geq \beta_c$, thus implying a logarithmic potential between heavy electrons,

$$P(R) \asymp \frac{1}{R^{\eta(T)}} , \quad (1)$$

$R \gg 1$ being the distance between test charges. The critical index $\eta(T)$ is known from the renormalization group (RG) analysis of Ref. [3] and equals $1/4$ at the critical point of the BKT transition in $2D$ XY , $\eta(T_c) = 1/4$. For $\beta < \beta_c$ one has instead

$$P(R) \asymp \exp[-R/\xi(t)] , \quad (2)$$

where $t = \beta_c/\beta - 1$, the correlation length goes as $\xi \sim e^{bt^{-\nu}}$, and the critical index ν is equal to $1/2$ in $2D$ XY . To determine the universality class of the finite-temperature $3D$ $U(1)$ LGT and to check if it coincides with that of $2D$ XY , the indices η and ν must be calculated and compared with $1/4$ and $1/2$, respectively ¹.

On the analytical side, Parga studied the $3D$ $U(1)$ LGT in the Lagrangian formulation [5], explaining that at high temperatures the system becomes effectively two-dimensional and, in particular, the monopoles of the original $U(1)$ gauge theory become

¹If not otherwise specified, from now on η will stand for $\eta(T_c)$.

vortices of the $2D$ system. The partition function turns out to coincide (at the leading order of the high-temperature expansion) with that of the $2D$ XY model in the Villain representation, thus supporting the Svetitsky-Yaffe conjecture. In the RG study of Refs. [1, 6, 7], high-temperature and dilute monopole gas approximations were used for the Villain formulation of finite-temperature $3D$ $U(1)$, which helped to derive a scaling law for the effective coupling between Polyakov loops with the lattice spacing. The resulting RG equations were shown to converge rapidly with the iteration number to the RG equations of the $2D$ XY model. This represents a strong indication that, indeed, the nature of the phase transitions in the two models is the same. Moreover, since the scaling with the lattice spacing coincides in the two cases, one concludes that the critical index ν is the same in the two models. The critical points and the index η in the Villain formulation of finite-temperature $3D$ $U(1)$ have been determined for various values of N_t in [7], via numerical analysis of the RG equations, confirming that $\eta = 1/4$.

On the numerical side, evidences that the deconfinement transition in the $3D$ $U(1)$ LGT belongs to the universality class of the $2D$ XY model come from our study of the phase transitions in $3D$ $Z(N)$ LGT [8, 9, 10, 11] for large values of N . These models exhibit two separate BKT-like phase transitions at critical couplings, say, $\beta_2(N)$ and $\beta_1(N)$, with $\beta_2(N) > \beta_1(N)$. While $\beta_2(N)$ diverges with N , the critical coupling $\beta_1(N)$ seems to converge to a value, which is expected, on general grounds, to represent the critical point of the $3D$ $U(1)$ LGT. Moreover, it turns out the critical indices at the transition in $\beta_1(N)$ do not depend on N in the interval $N \in [5 - 20]$ and are compatible with the universality class of the $2D$ XY model. The same phenomenon occurs in the $2D$ $Z(N)$ spin models for large N . This strongly suggests that $3D$ $U(1)$ near criticality belongs to the universality class of the $2D$ XY model.

The first direct simulation of $3D$ $U(1)$ on the lattice was performed on $L^2 \times N_t$ lattices with $L = 16, 32$ and $N_t = 4, 6, 8$ in [12]. These authors confirmed the expected BKT nature of the phase transition in the gauge model, but reported a critical index almost three times larger than that predicted for the $2D$ XY model, $\eta \approx 0.78$. The more recent analytical and numerical study of Ref. [13] indicated that, at least in the limit of vanishing spatial coupling, $\beta_s \rightarrow 0$, *i.e.* on an anisotropic lattice with decoupled space-like plaquettes, the $3D$ $U(1)$ LGT exhibits the same critical behavior of the $2D$ XY spin model. Numerical simulations of the isotropic model, on lattices with $N_t = 8$ and spatial extension up to $L = 256$, revealed instead that $\eta \approx 0.49$, which is still far from the XY value [14], thus leaving open the question about the universality class of $3D$ $U(1)$ LGT with nonvanishing β_s .

A possible explanation for this mismatch at large N_t can reside in the infinite order nature of the BKT transition. Indeed, the exponential divergence of the correlation length in a BKT transition implies a very slow, logarithmic convergence to the thermodynamic limit in the vicinity of the transition. As a consequence, very large volumes are required in order to enter the scaling region and safely extract critical indices.

In [9, 10] we developed a strategy based on duality, according to which $Z(N)$ LGTs

were mapped into spin models. This opened the way to the use of cluster algorithms, which facilitated the access to larger volumes, thus improving the description of criticality. In the present work we adjust that strategy to the study of the critical behavior of the $3D$ $U(1)$ LGT. This goes through the following steps:

- Calculate the disorder operator in the dual formulations of the $2D$ XY spin model and in the $3D$ $U(1)$ LGT in the dilute gas approximation.
- Derive and study the RG equations in the Villain formulation; compute the critical indices ν and η and give an analytical prediction for the index r related to the leading logarithmic correction.

This is the subject of Section 2. The main conclusion of these analytical investigations is that the critical behavior of the disorder operator of the $3D$ $U(1)$ LGT is governed by the critical behavior of the corresponding operator in the $2D$ XY model. Therefore, the universality problem can be studied with the help of the disorder operator on the same theoretical footing as with the help of the Polyakov loop correlation function.

- Use the cluster algorithm in the dual formulation of $3D$ $U(1)$ to determine the second moment correlation length and to locate the critical points of the deconfinement transition; then, compute the critical indices from the large distance behavior of the disorder operator.

This is done in Section 3.

In Section 4 we summarize our results and draw our conclusions.

2 Dual of $3D$ $U(1)$ LGT and disorder operator

We work on a periodic $3D$ lattice $\Lambda = L^2 \times N_t$ with spatial extension L and temporal extension N_t . With the goal of performing a RG analysis, we introduce anisotropic dimensionless couplings as

$$\beta_t = \frac{1}{g^2 a_t} , \quad \beta_s = \frac{\xi}{g^2 a_s} = \beta_t \xi^2 , \quad \xi = \frac{a_t}{a_s} , \quad (3)$$

where a_t (a_s) is lattice spacing in the time (space) direction, g^2 is the continuum coupling constant with dimension of an inverse length and $\beta = a_t N_t$ is the inverse temperature. The dual of $3D$ $U(1)$ LGT is given by

$$Z_\Lambda(\beta_n) = \sum_{\{r(x)\}=-\infty}^{\infty} \prod_x \prod_{n=1}^3 I_{r(x)-r(x+e_n)}(\beta_n) . \quad (4)$$

Here $I_r(x)$ is the modified Bessel function of first kind and order r , $\beta_3 = \beta_s$ and $\beta_1 = \beta_2 = \beta_t$. When $\beta_s = 0$ and $N_t = 1$ the theory reduces to the dual of the 2D XY model.

The conventional disorder operator in the XY model,

$$D(x, y) = \langle \exp [ic(r(x) - r(y))] \rangle, \quad 0 < c < 2\pi, \quad (5)$$

defines the free energy of the vortex-antivortex pair. It obeys the following bound [15]

$$D(x, y) \leq |x - y|^{-c^2\gamma(\beta)}, \quad \gamma(\beta) > 0. \quad (6)$$

A similar disorder operator in the 3D $U(1)$ LGT gives the free energy of a monopole-antimonopole pair. In this paper we use the following generalization of the disorder operator

$$D(x, y) = \langle \exp \left[i \frac{c}{N_t} \sum_{x_3=0}^{N_t} (r(x) - r(y)) \right] \rangle. \quad (7)$$

The reason for such definition will be explained shortly.

We want to calculate the disorder operators (5), (7) and derive RG equations from an effective coupling which describes the behavior of these operators near criticality. For simplicity we give details of the derivation for the XY model and then explain how it can be extended to $U(1)$ LGT at finite temperature. When both couplings are large, it is customary to use the Villain approximation, *i.e.*

$$I_r(x)/I_0(x) \approx \exp \left(-\frac{1}{2x} r^2 \right). \quad (8)$$

The Villain model, obtained by taking the approximation (8), is generally accepted to have the same universal properties as the original model [1, 5].

Let us consider first the general disorder operator

$$D(s) = \langle \exp \left[i \sum_x s(x) r(x) \right] \rangle, \quad (9)$$

where $s(x)$ are sources for dual variables $r(x)$. Substituting (8) into the partition function (4), we use the Poisson summation formula to perform the summation over the variables $r(x)$. In the case of the XY model, the disorder operator (9) is factorized in the product of the spin wave contribution and the contribution from the vortex configurations ($\beta_s = 0$, $\beta_t = \beta$),

$$D(s) = D_{\text{sw}}(s) D_v(s). \quad (10)$$

The spin wave contribution is given by (sum over repeated coordinates is understood)

$$D_{\text{sw}}(s) = \exp \left[-\frac{1}{4} \beta \sum_{x,y} s(x) G_{x,y} s(y) \right], \quad (11)$$

while the vortex part can be presented as

$$D_v(s) = D_v^{-1}(s=0) \sum_{\{m_x\}} \exp \left[-\pi^2 \beta m(x) G_{x,y} m(y) - \pi \beta m(x) G_{x,y} s(y) \right] . \quad (12)$$

Here, $G_{x,y} \equiv G_{|x-y|}$ is the two-dimensional massless Green's function. Our following calculations are based on the dilute gas approximation, which can be used when β is large enough. In this case the leading contribution comes from the configurations $m(x) = 0, \pm 1$ and, taking into account the neutrality of the vortex ensemble, we obtain after a long but standard algebra

$$D_v(s) \approx \exp \left[2\pi^3 \beta^2 \int_1^\infty r^3 e^{-2\pi^2 \beta D(r)} dr s(x) G_{x,y} s(y) \right] , \quad (13)$$

where $D(r) = G_0 - G_r$. Taking the asymptotics of the $D(r)$ function and combining the last equation with Eq. (11), we write down the result in the form

$$D(s) = \exp \left[-\frac{1}{4} \beta_{\text{eff}} s(x) G_{x,y} s(y) \right] , \quad (14)$$

where

$$\beta_{\text{eff}} = \beta - 2\pi^3 \beta^2 y^2 \int_1^\infty r^{3-2\pi\beta} dr , \quad (15)$$

and we have introduced the vortex activity as

$$y = 2 e^{-\frac{1}{2}\pi^2 \beta} . \quad (16)$$

The RG equations can be derived from the expression for β_{eff} , by integrating in Eq. (15) between the length scales a and $a + \delta a$, see *e.g.* [16]. Denoting $t = \ln a$, one finds

$$\frac{d\beta}{dt} = -2\pi^3 y^2 \beta^2 , \quad \frac{dy}{dt} = y (2 - \pi\beta) . \quad (17)$$

These equations coincide with the conventional RG equations for the XY model. This should not come as a surprise, because β_{eff} derived above equals the corresponding effective coupling for the spin-spin correlation and for the twist free energy up to $\mathcal{O}(y^4)$ [16].

To extend this result to the finite-temperature $U(1)$ LGT, we use the disorder operator (7) and calculate it in the anisotropic model (4). The calculations follow closely the ones for the twist free energy [7] and lead to the same expression, again up to $\mathcal{O}(y^4)$, for the β_{eff} that controls the behavior of the twist (see formula (24) in Ref. [7]). Hence, all the analysis of [7] remains valid for the disorder operator (7). In particular, the fixed point of RG equations scales with N_t as

$$\beta_t^f = \frac{2}{\pi} N_t . \quad (18)$$

An important consequence, relevant for our study, concerns the fall-off of the two-point disorder operator and the corresponding second-moment correlation length. Taking the sources in (9) in the form

$$s(z) = \frac{c}{N_t} (\delta_{z,x} - \delta_{z,y}) , \quad (19)$$

we find the leading term to be

$$D(x,y) = \frac{\text{const}}{R^\eta} , \quad R = |x - y| . \quad (20)$$

The index η for all N_t is found to be

$$\eta(\beta_{\text{eff}}) = \frac{c^2}{2\pi N_t} \beta_{\text{eff}} . \quad (21)$$

Since at the critical point β_{eff} takes the fixed point value (18), we finally obtain the expression for the index η at the phase transition point,

$$\eta = \left(\frac{c}{\pi} \right)^2 . \quad (22)$$

In particular, it leads to $\eta = 1/4$ when $c = \pi/2$, *i.e.* the value that equals the conventional value obtained from the spin-spin correlation function. The leading logarithmic correction to the power-like fall-off can also be easily computed at the critical point following the standard scheme (see, for instance, Section 4 of [17]),

$$D(x,y) = \frac{\text{const}}{R^{\eta_c}} (\ln R)^{2r} , \quad (23)$$

where

$$r = - \left(\frac{c}{2\pi} \right)^2 . \quad (24)$$

Note that, r being negative, the leading logarithm appears in the denominator, in contrast to the spin-spin correlation function.

The above observations imply that such RG-invariant quantities, like the second-moment correlation length and the Binder cumulant, take universal values that are independent of N_t and are known for the XY model [18, 19].

In the next Section we combine this observation with a cluster algorithm to locate critical points for different N_t and to compute the index η .

3 Numerical data

In this Section we simulate the dual $3D$ $U(1)$ model on a $L^2 \times N_t$ lattice, using the cluster algorithm described in [20]. We define the dual second-moment correlation length ξ_2 in the following way:

$$\xi_2 = \frac{\sqrt{\frac{X}{F} - 1}}{2 \sin \pi/L} , \quad (25)$$

Table 1: Values of the A_c and B_c constants in the scaling of ξ_2 , Eq. (26).

c	A_c	B_c
$\pi/3$	1.166...	0.307...
$\pi/2$	0.751...	0.212...
$2\pi/3$	0.533...	0.168...

$$\chi = \left\langle \sum_{x,y} D(x,y) \right\rangle, \quad F = \left\langle \sum_{x,y} e^{2\pi i(x_1-y_1)/L} D(x,y) \right\rangle,$$

where $D(x,y)$ is the disorder operator defined in (7) and c in that equation is an arbitrary parameter, defining the numerical value of ξ_2 . In what follows we will consider $c = \pi/3, \pi/2$ and $2\pi/3$.

From the analytical expression for the correlation function, one can find the following scaling of ξ_2 with L at the critical point, using the method described in [18, 19]:

$$\frac{\xi_2}{L} = A_c - \frac{B_c}{\ln L + C}. \quad (26)$$

The values A_c and B_c are calculated in spin wave approximation, taking $\beta_{\text{eff}} = 2/\pi$. In that case

$$\begin{aligned} \chi(\beta) &= \sum_R \exp\left(-\frac{c^2\beta}{2}D(R)\right), \\ F(\beta) &= \sum_R \exp\left(-\frac{c^2\beta}{2}D(R)\right) \cos \frac{2\pi x}{L}, \\ A_c &= \lim_{L \rightarrow \infty} \xi_2(\beta_{\text{eff}}), \\ B_c &= \frac{1}{\pi} \lim_{L \rightarrow \infty} \left. \frac{d\xi_2(\beta)}{d\beta} \right|_{\beta=\beta_{\text{eff}}}, \end{aligned} \quad (27)$$

where the sum is taken over the whole lattice and $D(R)$ is the Green's D function calculated on a lattice of size L , which can be written as a one-dimensional sum over momentum variables. For the different values of c we calculate the values of A_c and B_c using (27). Results are given in Table 1.

To extract the critical point from the scaling of ξ_2 with L , we use the following two methods:

- At each fixed value of β , perform the fit to $\xi_2(\beta, L)$ with

$$\frac{\xi_2(\beta, L)}{L} = A(\beta) - \frac{B(\beta)}{\ln L + C(\beta)}, \quad (28)$$

Table 2: Values of β_c obtained for $c = \pi/3, \pi/2$ and $2\pi/3$ for $N_t = 1, 2, 4, 8$, by the two different fit methods described in the text.

N_t	c	fit A, B fixed	fit B, A fixed
1	$\pi/3$	1.1185(36)	1.1184(9)
	$\pi/2$	1.1195(2)	1.1192(2)
	$2\pi/3$	1.1176(2)	1.1190(6)
2	$\pi/3$	1.84415(15)	1.84413(10)
	$\pi/2$	1.8460(8)	1.8458(7)
	$2\pi/3$	1.8487(8)	1.8488(7)
4	$\pi/3$	2.991(28)	2.991(23)
	$\pi/2$	3.005(14)	3.010(9)
	$2\pi/3$	3.027(9)	3.032(8)
8	$\pi/3$	5.567(7)	5.565(18)
	$\pi/2$	5.572(8)	5.573(12)
	$2\pi/3$	5.627(20)	5.635(18)

Table 3: Expected critical indices η and r , defined by (22) and (24).

c	η	r
$\pi/3$	$1/9 = 0.111\dots$	$-1/36 = -0.028\dots$
$\pi/2$	$1/4 = 0.25$	$-1/16 = -0.0625$
$2\pi/3$	$4/9 = 0.444\dots$	$-1/9 = -0.111\dots$

fixing $B(\beta)$ to the known value B_c ; β_c is then found as the point where $A(\beta) = A_c$ (see Fig. 1, left panels).

- The same procedure with $A(\beta)$ fixed to the known value A_c and β_c found as the point where $B(\beta) = B_c$ (see Fig. 1, right panels).

Results are summarized in Table 2. We note that the values for β_c obtained for $N_t = 8$ are much larger than our previous determination in the standard formulation of $3D$ $U(1)$ [14], $\beta_c = 3.06(11)$, thus explaining why we found there $\eta \approx 0.49$, a value far away from the expected $1/4$.

Measuring the correlation function as a function of R in the vicinity of the critical points allowed to perform the fit to the function $\Gamma(R) = A/2 \exp(-\pi\eta D(R))(\ln(R+1)^{2r} + \ln(L-R+1)^{2r})$. The expected values for η and r , given by (22) and (24), are summarized in Table 3.

Since the correlation function values for different R are obtained from the same set of measurements, these data cannot be used as independent points in a standard fit procedure. Indeed, the procedure we used has been the following:

- take the correlation function values for $R = 25 - 50$ for $L = 256$ and $R = 50 - 100$ for $L = 512$;
- calculate the covariance matrix $W(R_1, R_2) = \langle \Gamma(R_1) \Gamma(R_2) \rangle - \langle \Gamma(R_1) \rangle \langle \Gamma(R_2) \rangle$ using the jackknife algorithm;
- diagonalize W , obtaining the eigenvalues $\lambda(R')$ and the corresponding transformation matrix $V(R', R)$ ($R_{\min} \leq R' \leq R_{\max}$). Since the covariance matrix is symmetric, $V(R, R')$ is orthogonal, so that $W(R_1, R_2) = \sum_{R'=R_{\min}}^{R_{\max}} V(R', R_1) \lambda(R') V(R', R_2)$;
- perform a change of basis, obtaining $p(R') = \sum_{R=R_{\min}}^{R_{\max}} V(R', R) \Gamma(R)$. Since the covariance matrix in the new basis is diagonal, the new $P(R')$ variables are independent.
- make a fit to the points $p(R')$ with weights $1/\lambda(R')$ (meaning that we minimize $\chi^2 = \sum_{R'=R_{\min}}^{R_{\max}} (\Delta p(R'))^2 / \lambda(R')$).

Results are given in Tables 4 and 5.

Alternative ways to obtain η are (i) through the scaling with L of the susceptibility of the magnetization at the critical point, $\chi = AL^{-\gamma/\nu}$, using the formula $\eta = 2 - \gamma/\nu$, or (ii) through the scaling with L of the magnetization at the critical point, $M = AL^{-\beta/\nu}$, using the formula $\eta = 2\beta/\nu$, which assumes the validity of the hyperscaling relation $d = 2\beta/\nu + \gamma/\nu = 2$. Once we set c in (5) to be $2\pi/K$, we can build two equally acceptable definitions of the magnetization: the *standard* magnetization,

$$M_L = \langle |M| \rangle, \quad M = \sum_x \exp \left(i \frac{c}{N_t} r_x \right),$$

and the *rotated* one,

$$M_R = \left\langle \frac{\text{Re } M^{KN_t}}{|M^{KN_t}|} |M| \right\rangle.$$

It turns out that the hyperscaling relation is better satisfied if β/ν is extracted from the standard magnetization M_L and γ/ν from the susceptibility of the rotated magnetization χ_{M_R} , with respect to other possible combinations (see Table 6 for a comparison in some selected cases; here, $d_{M_*, \chi_{M_*}}$, with $*$ equal to L or R , stands for the value of d obtained by the hyperscaling formula $d = 2\beta/\nu + \gamma/\nu$ when β/ν is extracted from the scaling of the magnetization M_* and γ/ν from that of the magnetization susceptibility χ_*).

In Table 7 we summarize our determinations of the critical indices β/ν , as obtained from the scaling of the standard magnetization, γ/ν , as obtained from the scaling of the rotated magnetization susceptibility, $\eta = 2 - \gamma/\nu$, as well as $d_{M_L, \chi_{M_R}}$, for $N_t = 1, 2, 4$ and 8.

Finally, we constructed the continuum limit fitting the critical couplings $\beta_{t,c}$ from Table 2 using several dependences on N_t . As estimate of the critical point we took the

half-sum of the largest and of the smallest values obtained, for a given N_t , considering the three possible choices for c and the two fitting methods; as estimate of its uncertainty, we took the half-difference of the same values. The best fit is given by the function $\beta_{t,c} = 0.772(90) + 0.600(29)N_t - 0.252(64)/N_t$, $\chi^2 = 9.13$ (see Fig. 2).

4 Summary

In this paper we have studied the disorder operator (7) in the dual formulation of the finite-temperature $3D$ $U(1)$ LGT. We obtained and analyzed the RG equations in the Villain formulation of the model. These equations describe the critical behavior across the deconfinement phase transition. The Wilson formulation in its dual representation has been studied via numerical simulations for three values of the constant c entering the definition (7). Our main findings can be shortly summarized as follows.

- We have calculated analytically the critical indices η and r for any values of c , using the conventional RG. We have found that the critical behavior of the disorder operator of the $3D$ $U(1)$ gauge theory is governed by the critical behavior of the corresponding operator in the $2D$ XY model. It is important to stress that the index r , describing the leading logarithmic correction, is negative for the disorder operator.
- Using a cluster algorithm, we have simulated the dual form of the $3D$ $U(1)$ LGT, computed the disorder operator, the second moment correlation length, the standard magnetization and the rotated magnetization of the dual variables for three values of the constant $c = \pi/3, \pi/2$ and $2\pi/3$. In this way we have located critical points of the finite-temperature model for $N_t = 1, 2, 4$ and 8 , computed the critical indices η and r and checked the hyperscaling relation.
- We have computed the critical points for several temporal extensions N_t . In the continuum limit we have found $T_c \approx 0.600g^2$. This value agrees with the value obtained in [7] from the study of the critical behavior of the twist free energy.

It is important to stress that, while the index η agrees reasonably with analytical and universality predictions for all values of c , this is not always the case for the index r . However, we would like to stress that both RG study and numerical simulations show that this index is negative. This is an interesting property of the disorder operator.

Our final remark concerns the check of the hyperscaling relation. Table 6 shows the value of d extracted in four possible ways. One can conclude that the hyperscaling relation is satisfied only when β/ν is calculated from the conventional magnetization and γ/ν from the susceptibility of the rotated magnetization. This remains true for all values of c and might indicate some important property of the disorder operator which we miss at the moment.

Summarizing, we would like to stress that this work, together with our previous studies, leaves little doubt, if any, that the deconfinement phase transition in finite-temperature $3D$ $U(1)$ LGT belongs to the universality class of the $2D$ XY model, thus supporting the Svetitsky-Yaffe conjecture.

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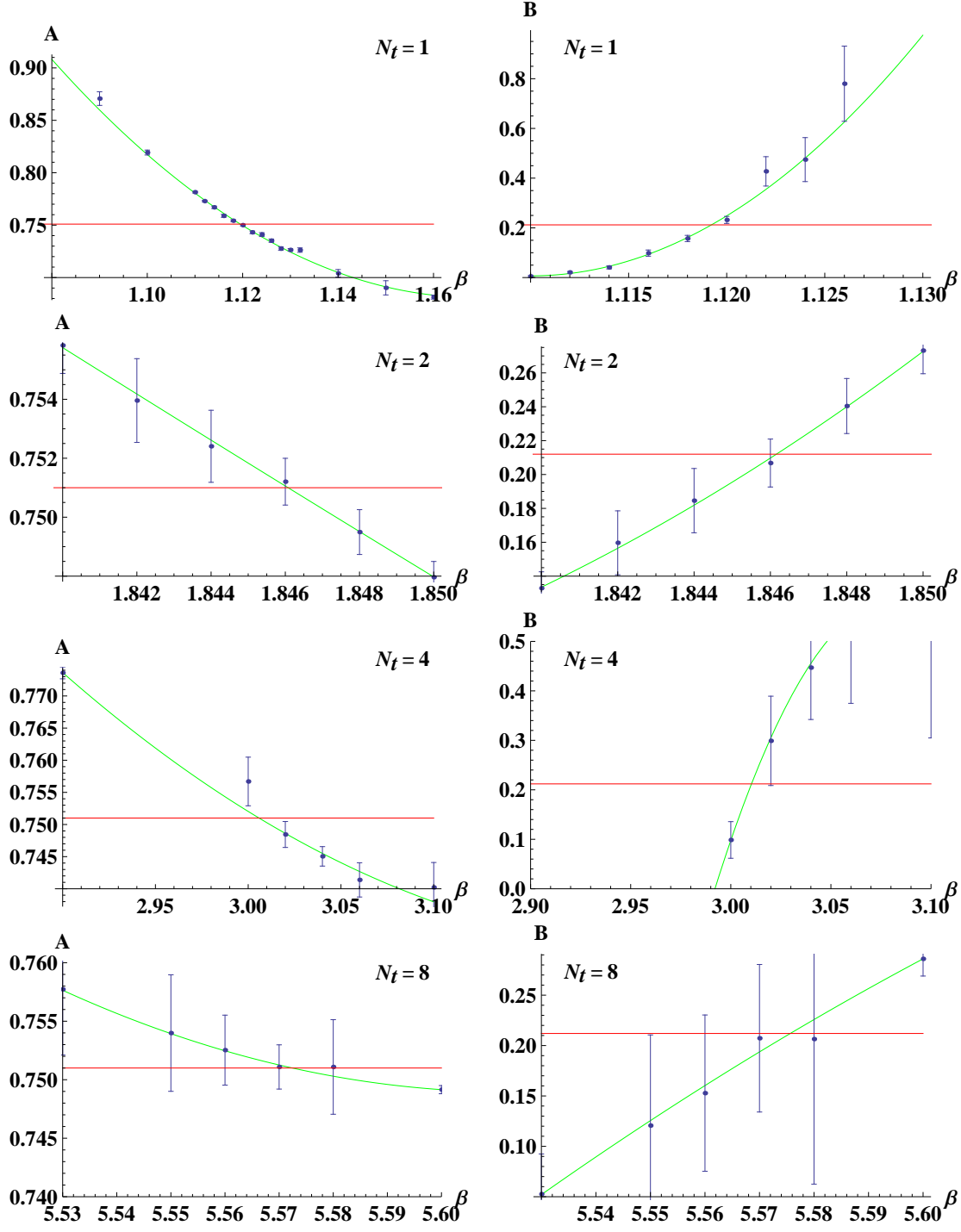


Figure 1: Determination of β_c for $N_t = 1, 2, 4$ and 8 from fits of the scaling (28) for $A(\beta)$ keeping $B(\beta)$ fixed at B_c (left panels) and for $B(\beta)$ keeping $A(\beta)$ fixed at A_c (right panels) in the case $c = \pi/2$.

Table 4: η values obtained from fitting the correlation function dependence on R to $\Gamma(R) = A/2^a \exp(-\pi\eta D(R))[\ln(R+1)^{2r} + a \ln(L-R+1)^{2r}]$ for $N_t = 1$ and 2; for each L , the first line corresponds to the fit with r fixed at zero, the third to the fit with $a = 0$.

N_t, c, β_c	L	A	η	r	χ_r^2
$N_t = 1$ $c = \pi/3$ $\beta_c = 1.1195$	256	0.97236(72)	0.11820(16)	0	8.51
		1.0123(36)	0.11107(64)	-0.0260(23)	1.43
		0.9602(15)	0.1031(19)	-0.0256(31)	2.27
	512	0.96850(57)	0.11756(11)	0	5.62
		0.9859(34)	0.11576(36)	-0.0091(18)	3.71
		0.96698(41)	0.1000(21)	-0.0348(41)	2.28
$N_t = 1$ $c = \pi/2$ $\beta_c = 1.1195$	256	0.9562(16)	0.26495(37)	0	7.28
		1.0532(69)	0.2476(13)	-0.0628(44)	0.88
		0.9264(33)	0.2256(43)	-0.0670(73)	1.64
	512	0.9477(13)	0.26361(27)	0	5.56
		1.0035(79)	0.25819(77)	-0.0287(39)	2.74
		0.94626(74)	0.2145(46)	-0.0983(91)	1.66
$N_t = 1$ $c = 2\pi/3$ $\beta_c = 1.1195$	256	0.9818(30)	0.46843(67)	0	4.77
		1.176(22)	0.4350(40)	-0.119(13)	1.35
		0.9251(94)	0.392(13)	-0.131(22)	1.96
	512	0.9651(27)	0.46607(54)	0	4.39
		1.098(20)	0.4545(18)	-0.0635(92)	2.33
		0.9632(19)	0.361(14)	-0.210(27)	2.02
$N_t = 2$ $c = \pi/3$ $\beta_c = 1.8460$	256	0.98785(42)	0.117439(95)	0	3.03
		1.0067(32)	0.11405(57)	-0.0122(20)	1.25
		0.9812(11)	0.1096(12)	-0.0130(22)	1.22
	512	0.98532(34)	0.116996(67)	0	2.89
		0.9971(30)	0.11579(31)	-0.0061(15)	2.21
		0.98473(38)	0.1110(21)	-0.0119(42)	2.52
$N_t = 2$ $c = \pi/2$ $\beta_c = 1.8460$	256	0.97623(94)	0.26360(21)	0	2.41
		1.0193(86)	0.2559(15)	-0.0279(55)	1.23
		0.9616(29)	0.2455(36)	-0.0306(60)	1.19
	512	0.97150(76)	0.26281(15)	0	2.20
		0.9951(77)	0.26046(77)	-0.0122(39)	1.88
		0.97069(83)	0.2510(56)	-0.023(11)	2.06
$N_t = 2$ $c = 2\pi/3$ $\beta_c = 1.8460$	256	0.9704(21)	0.46677(47)	0	2.32
		1.053(28)	0.4523(49)	-0.053(17)	1.79
		0.9416(92)	0.429(12)	-0.064(21)	1.70
	512	0.9640(16)	0.46585(31)	0	2.09
		1.024(18)	0.4602(16)	-0.0301(86)	1.71
		0.9626(16)	0.428(14)	-0.075(29)	1.87

Table 5: Same as Table 4 for $N_t = 4$ and 8.

N_t, c, β_c	L	A	η	r	χ_r^2
$N_t = 4$ $c = \pi/3$ $\beta_c = 3.005$	256	1.00035(34)	0.114548(72)	0	1.70
		0.9952(41)	0.11543(70)	0.0033(26)	1.66
		1.0033(12)	0.1181(14)	0.0061(24)	1.40
	512	0.99975(40)	0.114423(76)	0	2.19
		0.9943(28)	0.11495(28)	0.0028(14)	2.07
		0.99998(45)	0.1170(23)	0.0051(45)	2.18
$N_t = 4$ $c = \pi/2$ $\beta_c = 3.005$	256	1.00162(84)	0.25785(18)	0	1.64
		0.991(11)	0.2596(18)	0.0067(69)	1.64
		1.0078(34)	0.2656(41)	0.0132(69)	1.48
	512	1.0017(12)	0.25781(23)	0	2.97
		0.9971(93)	0.25825(92)	0.0023(47)	3.02
		1.0012(13)	0.2521(72)	-0.011(14)	3.00
$N_t = 4$ $c = 2\pi/3$ $\beta_c = 3.005$	256	1.0045(15)	0.45857(31)	0	1.26
		0.978(25)	0.4629(41)	0.017(16)	1.25
		1.0164(75)	0.4738(94)	0.026(16)	1.18
	512	1.0025(21)	0.45808(39)	0	2.09
		0.993(18)	0.4589(17)	0.0046(88)	2.12
		1.0023(23)	0.455(16)	-0.005(32)	2.13
$N_t = 8$ $c = \pi/3$ $\beta_c = 5.572$	256	0.99993(37)	0.113792(82)	0	1.83
		1.0010(45)	0.11360(81)	-0.0007(29)	1.90
		0.9989(16)	0.1126(17)	-0.0020(29)	1.87
	512	1.00055(34)	0.113899(65)	0	2.22
		1.0051(23)	0.11339(26)	-0.0024(12)	2.08
		1.00026(43)	0.1119(19)	-0.0040(37)	2.21
$N_t = 8$ $c = \pi/2$ $\beta_c = 5.572$	256	1.00008(84)	0.25608(19)	0	1.60
		1.005(11)	0.2553(19)	-0.0030(68)	1.66
		0.9971(37)	0.2526(42)	-0.0059(71)	1.62
	512	0.99982(81)	0.25599(15)	0	1.88
		1.0090(65)	0.25506(67)	-0.0047(33)	1.84
		0.99974(98)	0.2552(54)	-0.001(11)	1.92
$N_t = 8$ $c = 2\pi/3$ $\beta_c = 5.572$	256	1.0005(13)	0.45529(28)	0	0.97
		0.995(21)	0.4562(35)	0.003(13)	1.01
		1.0003(69)	0.4550(84)	0.000(14)	1.01
	512	0.9998(21)	0.45507(40)	0	2.21
		0.997(20)	0.4553(19)	0.0014(97)	2.25
		1.008(23)	0.470(15)	0.030(30)	2.20

Table 6: Comparison of the results for the hyperscaling relation for different ways of obtaining β/ν and γ/ν (see the text for the explanation of the notation $d_{M_*,\chi_{M_*}}$).

N_t, c, β_c	L_{\min}	$d_{M_L, \chi_{M_L}}$	$d_{M_L, \chi_{M_R}}$	$d_{M_R, \chi_{M_L}}$	$d_{M_R, \chi_{M_R}}$
$N_t = 1$ $c = \pi/2$ $\beta_c = 1.1195$	16	1.824(20)	2.0205(22)	2.229(34)	2.425(16)
	32	1.858(14)	2.0169(17)	2.231(25)	2.389(13)
	64	1.8848(99)	2.0137(12)	2.231(16)	2.3595(70)
	128	1.9058(57)	2.0127(14)	2.235(12)	2.3418(79)
	192	1.9143(41)	2.0116(22)	2.234(12)	2.3312(96)
	256	1.9190(59)	2.0083(16)	2.248(19)	2.337(15)
$N_t = 2$ $c = 2\pi/3$ $\beta_c = 1.8460$	16	1.9194(84)	1.9989(12)	2.500(21)	2.580(14)
	32	1.9340(58)	1.9986(10)	2.489(16)	2.554(11)
	64	1.9449(41)	1.9989(11)	2.507(23)	2.561(20)
	128	1.9514(32)	1.9983(13)	2.550(32)	2.597(30)
	192	1.9555(33)	1.9992(15)	2.522(50)	2.566(48)
	256	1.9596(34)	1.9995(32)	2.587(63)	2.627(63)

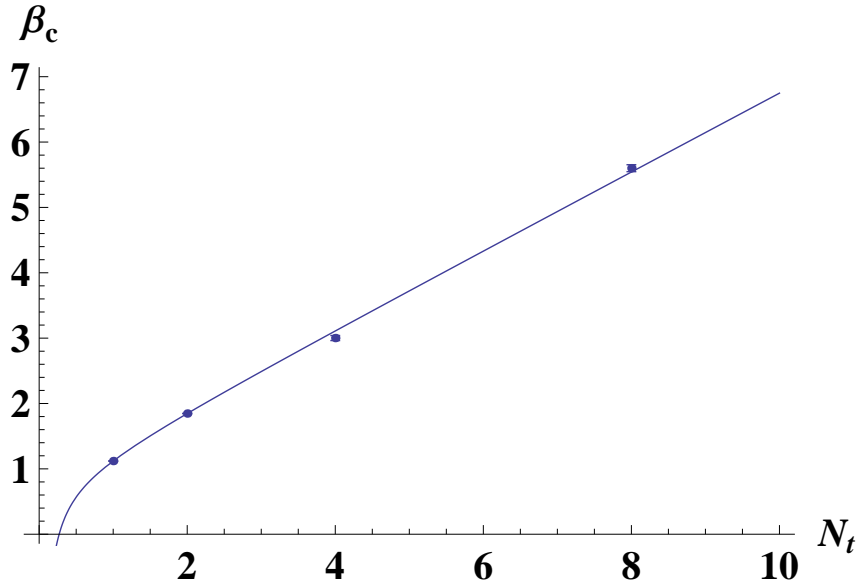


Figure 2: Fitting curve for the dependence on N_t of the critical couplings.

Table 7: Critical indices β/ν (from the standard magnetization), γ/ν (from the rotated magnetization susceptibility, $\eta = 2 - \gamma/\nu$ and $d_{M_L, \chi_{M_R}}$, for $N_t = 1, 2, 4$ and 8.

N_t, c, β_c	L_{\min}	β/ν	χ_r^2	γ/ν	χ_r^2	$d_{M_L, \chi_{M_R}}$	η
$N_t = 1$	128	0.059304(66)	2.14	1.8919(12)	1.32	2.0105(13)	0.1081(12)
$c = \pi/3$	192	0.059193(58)	0.75	1.8911(20)	1.73	2.0095(21)	0.1089(20)
$\beta_c = 1.1195$	256	0.05914(11)	1.07	1.8880(11)	0.23	2.0062(13)	0.1120(11)
$N_t = 1$	128	0.13323(14)	1.82	1.7463(12)	1.17	2.0127(14)	0.2537(12)
$c = \pi/2$	192	0.13299(13)	0.66	1.7456(19)	1.58	2.0116(22)	0.2544(19)
$\beta_c = 1.1195$	256	0.13288(24)	0.97	1.7425(11)	0.21	2.0083(16)	0.2575(11)
$N_t = 1$	128	0.23609(22)	1.15	1.5461(12)	1.11	2.0183(16)	0.4539(12)
$c = 2\pi/3$	192	0.23575(21)	0.48	1.5453(19)	1.38	2.0168(23)	0.4547(19)
$\beta_c = 1.1195$	256	0.23561(42)	0.81	1.54217(91)	0.13	2.0134(17)	0.45783(91)
$N_t = 2$	128	0.058718(47)	1.87	1.88167(65)	0.98	1.99910(74)	0.11833(65)
$c = \pi/3$	192	0.058652(58)	1.29	1.88256(96)	0.85	1.9999(11)	0.11744(96)
$\beta_c = 1.8460$	256	0.05862(12)	2.32	1.8831(18)	1.47	2.0003(21)	0.1169(18)
$N_t = 2$	128	0.131917(92)	0.70	1.7357(14)	2.77	1.9995(15)	0.2643(14)
$c = \pi/2$	192	0.131765(44)	0.081	1.7361(24)	4.04	1.9996(25)	0.2639(24)
$\beta_c = 1.8460$	256	0.131805(76)	0.11	1.7390(33)	3.47	2.0026(35)	0.2610(33)
$N_t = 2$	128	0.23416(20)	1.88	1.53001(93)	1.37	1.9983(13)	0.46999(93)
$c = 2\pi/3$	192	0.23390(27)	1.51	1.53140(97)	0.69	1.9992(15)	0.46860(97)
$\beta_c = 1.8460$	256	0.23381(56)	2.90	1.5319(21)	1.28	1.9995(32)	0.4681(21)
$N_t = 4$	128	0.057164(61)	2.05	1.88580(88)	1.87	2.0001(10)	0.11420(88)
$c = \pi/3$	192	0.05720(11)	2.82	1.8851(15)	2.37	1.9995(17)	0.1149(15)
$\beta_c = 3.005$	256	0.05726(20)	4.76	1.8832(19)	1.59	1.9977(23)	0.1168(19)
$N_t = 4$	128	0.12866(10)	1.06	1.74249(89)	1.67	1.9998(11)	0.25751(89)
$c = \pi/2$	192	0.12873(17)	1.36	1.7417(13)	1.84	1.9991(17)	0.2583(13)
$\beta_c = 3.005$	256	0.12884(31)	2.19	1.7397(11)	0.57	1.9974(17)	0.2603(11)
$N_t = 4$	128	0.22871(21)	1.38	1.5423(15)	4.02	1.9997(19)	0.4577(15)
$c = 2\pi/3$	192	0.22880(36)	1.94	1.5416(25)	5.61	1.9992(32)	0.4584(25)
$\beta_c = 3.005$	256	0.22898(69)	3.47	1.5388(41)	6.14	1.9967(55)	0.4612(41)
$N_t = 8$	128	0.056945(19)	0.24	1.88599(56)	0.82	1.99988(60)	0.11401(56)
$c = \pi/3$	192	0.056935(31)	0.32	1.88501(56)	0.34	1.99888(62)	0.11499(56)
$\beta_c = 5.572$	256	0.056882(22)	0.063	1.88589(53)	0.12	1.99965(57)	0.11411(53)
$N_t = 8$	128	0.128167(80)	0.56	1.74284(72)	0.85	1.99917(88)	0.25716(72)
$c = \pi/2$	192	0.12820(14)	0.79	1.74186(80)	0.52	1.9983(11)	0.25814(80)
$\beta_c = 5.572$	256	0.1279502(15)	$3.3 \cdot 10^{-5}$	1.7410(16)	0.71	1.9969(16)	0.2590(16)
$N_t = 8$	128	0.227822(65)	0.17	1.54418(53)	0.46	1.99982(66)	0.45582(53)
$c = 2\pi/3$	192	0.22778(11)	0.23	1.54502(31)	0.083	2.00059(53)	0.45498(31)
$\beta_c = 5.572$	256	0.227607(50)	0.021	1.54541(58)	0.097	2.00063(68)	0.45459(58)